$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial \xi}+\frac{\partial u}{\partial \eta},\quad \frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial \xi^2}+2\frac{\partial^2 u}{\partial \xi\partial \eta}+\frac{\partial^2 u}{\partial \eta^2},$$

$$\frac{\partial u}{\partial t}=a\left(\frac{\partial u}{\partial \xi}-\frac{\partial u}{\partial \eta}\right),\frac{\partial^2 u}{\partial t^2}=a^2\left(\frac{\partial^2 u}{\partial \xi^2}-2\frac{\partial^2 u}{\partial \xi\partial \eta}+\frac{\partial^2 u}{\partial \eta^2}\right).$$

Jednačina (1) u novim promjenljivim ima oblik $\frac{\partial^2 u}{\partial \xi\partial \eta}=0$. Proizvoljno rješenje ove jednačine je $u=F_1(\xi)+F_2(\eta)$, gdje su $F_1$ i $F_2$ proizvoljne dva puta neprekidno-diferencijbilne funkcije. Saglasno uvedenim smjenama, jednačina (1) ima rješenje

$$u=F_1(x+at)+F_2(x-at). \tag{3}$$

Da bismo odredili nepoznate funkcije $F_1$ i $F_2$ iskoristićemo početne uslove (2):

$$u\big|_{t=0}=F_1(x)+F_2(x)=\varphi(x),\qquad \frac{\partial u}{\partial t}\Big|_{t=0}=aF_1'(x)-aF_2'(x)=\psi(x).$$

Poslije integracije druge jednakosti dobijamo

$$\begin{cases} F_1(x)+F_2(x)=\varphi(x) \\ F_1(x)-F_2(x)=\dfrac{1}{a}\displaystyle\int_{x_0}^{x}\psi(z)dz+C, \end{cases} \tag{4}$$

gdje su $x_0$ i $C$ proizvoljne konstante. Iz sistema (4) nalazimo

$$F_1(x)=\frac{\varphi(x)}{2}+\frac{1}{2a}\int_{x_0}^{x}\psi(z)dz+\frac{C}{2}\ \text{ i }\ F_2(x)=\frac{\varphi(x)}{2}-\frac{1}{2a}\int_{x_0}^{x}\psi(z)dz-\frac{C}{2}. \tag{5}$$

Zamjenom (5) u (3) dobijamo da je

$$u=\frac{\varphi(x+at)+\varphi(x-at)}{2}+\frac{1}{2a}\left(\int_{x_0}^{x+at}\psi(z)dz-\int_{x_0}^{x-at}\psi(z)dz\right),$$

odnosno

$$u=\frac{\varphi(x+at)+\varphi(x-at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}\psi(z)dz. \tag{6}$$

Formula (6) daje rješenje Kz (1)-(2) i naziva se Dalamberova formula.

**Primjer 1.** Naći rješenje jednačine

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial^2 x}\quad (-\infty<x<+\infty,\ \ t>0),u\big|_{t=0}=x^2,\ \frac{\partial u}{\partial t}\Big|_{t=0}=0.$$

Kako je $a$=1, $\varphi(x)=x^2$ i $\psi(x)=0$, to zamjenom u Dalamberovu formulu dobijamo da je $u=\frac{(x-t)^2+(x+t)^2}{2}$ ili $u=x^2+t^2$.

**Primjer 2.** Naći rješenje jednačine

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial^2 x}\quad (-\infty<x<+\infty,\ \ t>0),u\big|_{t=0}=0,\ \frac{\partial u}{\partial t}\Big|_{t=0}=x.$$